On synchronization for Sprott systems

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Abstract. In the paper we shown the results of three non-linear observers that solve the synchronization problem for a system proposed by Munmuangsaen et al. (2011). Each of these observers has particular properties. First, a non-linear observer with linear output injection is presented, which guarantees asymptotic stability. With this observer we can affect the damping of transient response of error convergence. Second, we present a Thau observer. The synchronization error of this observer is bounded. Finally, we present a sliding-mode observer. This observer has the property to bring the error dynamics to zero in finite-time. We perform two robustness tests in order to know the behaviour of each proposed observer under variations of parameters and disturbances. We provide results of numerical simulations as an illustration of error dynamics and convergence of trajectories of GS11 system-observer synchronization.
Keywords: Chaos synchronization, linear injection, observer, Thau observer, sliding-modes.

1 Introduction

Chaos synchronization phenomenon in a master-slave formalism has been intensively studied in the literature. Several synchronization methods using chaotic systems have been developed since the works by Pecora and Carol [1] in 1990, and by Cuomo et al. [2] in 1993. They constructed a master-slave scheme where the slave is a modified copy of the master system. Since the work by Nijmeijer and Mareels [3] the synchronization problem can be viewed from a perspective of control theory and solved employing non-linear observers. The idea is to use the output vector of the master system to control the slave system. Then we can synchronize the master system and slave system, this method turns out to be one of the most efficient for chaos synchronization [4],[5].

We can found some papers devoted to study the chaotic systems reported by Sprott [11] in the sense of solution of synchronization problem. The Sprott systems present a particular structure and have a practical application in secure communications. Since they can be implemented by simple circuits whose properties can be predicted and controlled with very high accuracy. Xiao-Dong et al. [12] present a terminal sliding-mode controller to realize finite-time synchronization of Sprott circuits. This controller renders the closed loop system robust with respect to bounded disturbances and parametric variations. A PID controller was developed via the EP (Evolutionary Programming) algorithm by Hsin-Chieh et al. [13], the performance criterion utilized is the IAE (Integrated Absolute Error). Using the EP algorithm, the optimal control gains of PID controller are derived such that the performance index of IAE is as minimal as possible. Bai et al. [14] described a straightforward technique that consists in adding control variables in the observer in order to obtain a linear error dynamics.

In the literature, we can find different types of observers used to synchronize chaotic systems and they are applied to secure communication systems. As an example, Wang and Ge [6] present an adaptive observer via backstepping design. Teh-Lu et al. [7] present an adaptive observer, it is constructed to synchronize the master system whose dynamics are subjected to the system's disturbances and/or some unknown parameters. Perruquetti et al. [5] present a sliding-mode observer for non-linear systems that can be transformed in a linear canonical form up to an output injection. Thau observer [8] is a model-based observer which reconstructs state variables of a class of nonlinear systems [9] An alternative of Thau observer is presented by Starkov et al. [10] and its extension for the case of systems possessing a positively invariant domain. Each observer has specific properties, it which are useful for secure communication systems based in a master-slave scheme. An adaptive observer estimates state variables and unknown parameters of the master system. Moreover, the sliding-mode observer has the property to bring the error dynamics to zero in finite-time.

Recently, Munmuangsaen et al. [15] present a jerk function, exhibiting a chaotic behaviour for different non-linear functions. This model is given by the following expression:

$$\dddot{x} + \ddot{x} + x = f(\dot{x}); \tag{1}$$

where $f(\dot{x})$ is the non-linear function required for chaos. To find chaotic solutions, Munmuangsaen et al. [15] employed a numerical search procedure and have been found twelve non-linear functions (labeled by 'GS1' to 'GS12'), listed in Table 1.

Table 1. Functions $f(\dot{x})$ that produces chaos.

Case	$f(\dot{x})$
GS1	$\pm 0.1 \exp(\mp \dot{x})$
GS2	$\pm \exp(\mp \dot{x} - 2)$
GS3	$\pm 5.1 \cos(\pm \dot{x} + 0.1)$
GS4	$\pm 0.2 \tan(\mp \dot{x})$
GS5	$\pm sign(1 \mp 4\dot{x})$
GS6	$\pm \dot{x}^2 - 0.2\dot{x}^3$
GS7	$\pm \frac{1}{(\dot{x} \pm 2)^2}$
GS8	$-5\dot{x} \pm \lvert 1 \pm 5\dot{x} \rvert$
GS9	$\pm \frac{0.4}{\lvert \pm \dot{x} + 1 \rvert}$
GS10	$\pm \frac{1}{\lvert \pm \dot{x} + 1 \rvert^{0.5}}$
GS11	$\pm 4 sen(\pm \dot{x} + 1) - \dot{x}$
GS12	$\pm \cosh(\dot{x}) - 0.6\dot{x}$

The purpose of our paper is to present results of three non-linear observers design for GS systems from [15]. According with Moon [16] for a dynamical system to display chaotic behaviour it has to be either nonlinear. Commonly, the nonlinearity in chaotic systems are polynomial terms. We choose the GS systems because they present a different structure than classical chaotic systems. The GS systems have four linear terms (or five terms in two cases) and one non-linear term. The nonlinearity of GS system can be polynomial, exponential, trigonometric or rational function. Also, this systems are easy to construct with electronic components and scaling over a wide range of frequencies [11],[17],[18]. For all the above features, these system may be applied in future researches on the development of secure communication systems.

The paper is organized as follows. A non-linear observer with linear output injection is presented in Section 2. With this observer we can affect the damping of transient response, choosing an appropriate value of gain matrix. This observer guarantees asymptotic stability for all systems proposed by Munmuangsaen et al. (2011). In Section 3 we present a Thau observer design for GS11 system using inequality from [10]. Further, in Section 4, we present a sliding-mode observer for GS11 system which have the property to bring the error estimation to zero in finite-time. Conclusions are presented in Section 5.

2 Non-linear observer with linear output injection for GS systems

Observers with linear output injection can be used in dynamic systems whose non-linear part depends only upon the output, see [3]. The error of this observer has a linear dynamic and the convergence time of this observer can be tuned by the proper choice of the eigenvalues of the gain matrix.

Firstly, is necessary to present (1) in a state-space representation. Choosing state variables $x_1 = x$; $x_2 = \dot{x}$; $x_3 = \ddot{x}$; Eq. (1) can be expressed by the following state-space form:

$$\begin{aligned} \dot{x}_1 &= x_2; \\ \dot{x}_2 &= x_3; \\ \dot{x}_3 &= f(x_2) - x_1 - x_3. \end{aligned} \tag{2}$$

Since the non-linear term in (2) depends on x_2 then we choose $y = x_2$ as output vector in order to design the non-linear observer with linear output injection.

Also, the nonlinearity is not a smooth function in some cases, but this does not affect the design; this is because a dissipative chaotic flow has the property that trajectories are attached to a bounded region of state-space by strange attractor and its divergence is negative [19],[20]. Since (15) is dissipative with constant state-space contraction of -1. Then $|f(x_2)| \leq \gamma |x_2|$ form some constant $\gamma > 0$. That means the solutions for all initial conditions are well defined on $\Re^3$.

The non-linear observer with linear output injection for Eq. (2) proposed for GS systems is described by the following equations:

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + k_1(y - \hat{x}_2); \\ \dot{\hat{x}}_2 &= \hat{x}_3 + k_2(y - \hat{x}_2); \\ \dot{\hat{x}}_3 &= f(x_2) - \hat{x}_1 - \hat{x}_3 - k_3(y - \hat{x}_2); \end{aligned} \tag{3}$$

with $k_1; k_2; k_3 \in \Re$.

The observer error is defined as $e = x - \hat{x}$. Dynamics of the observer error is given by following equation:

$$\dot{e} = \begin{bmatrix} 0 & 1 - k_1 & 0 \\ 0 & -k_2 & 1 \\ -1 & -k_3 & -1 \end{bmatrix} e. \tag{4}$$

In order to guarantee than trajectories of estimated states converge to real state,we found a gain matrix $K = [k_1, k_2, k_3]$ such that error dynamics is asymptotically stable.

Choosing $\mu_i = -1, i = 1, 2, 3$, as desired poles in the left half of the complex plane, the gain matrix is $K = [-2.375, 3.5, 3.25]$.

Numerical simulation of synchronization between non-linear observer with linear output injection and GS system taking $f(x_2) = 4\sin(x_2 + 1) - 2.2x_2$, are shown in Figs. 1 and 2. The initial conditions of the two systems are different, the initial condition of (1) is $x_i(0) = 0$ ($i = 1, 2, 3$), the initial condition of (3) is $\hat{x}_i(0)$ ($i = 1, 2, 3$).

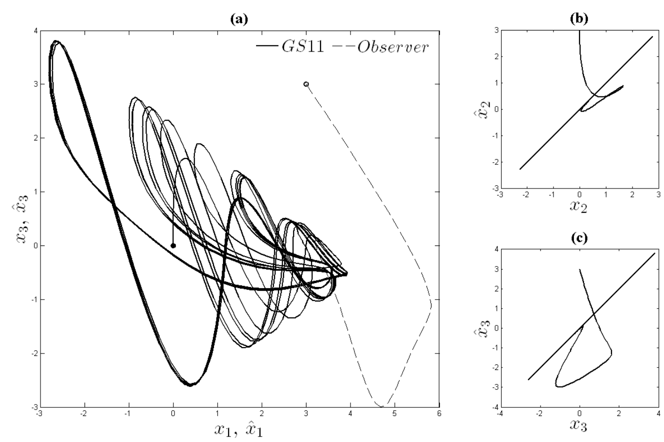

Fig. 1. Synchronization of GS systems:(a) Comparison of dynamics of GS system and non-linear observer with liner output injection, (b-c) Synchronization between real states and estimated states.

We can see that trajectories of non-linear observer with linear output injection converge to chaotic attractor of GS system (Fig. 1(a)) and that estimated states are synchronized with real states (Fig. 1(b-c)). The error dynamics of master-slave synchronization

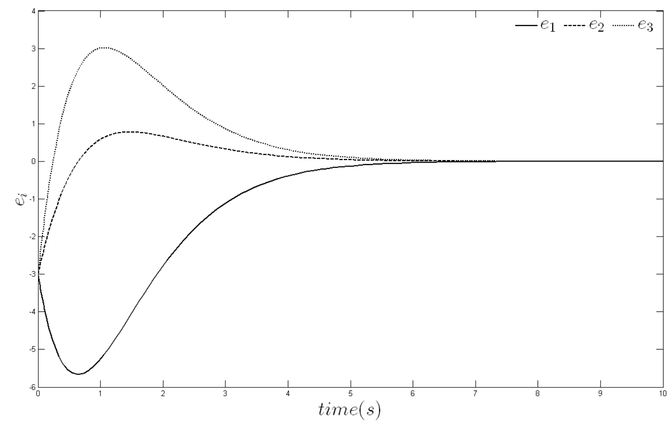

Fig. 2. Error dynamics of master-slave synchronization of non-linear observer with linaer output injection and GS system considering $f(x_2) = 4\sin(x_2 + 1) - 2.2x_2$.

converge to zero in approximately 8 seconds, as it is shown in Fig. 2. If we desire to speed up the convergence time is necessary to choose an appropriate gain matrix.

3 Thau observer design for GS11 system

In this section we present a Thau observer design for GS11 system, where the non-linear functions is defined as $f(x_2) = b\sin(x_2 + 1) - cx_2$. We consider the case GS11 because is a Lipschitz function.

Thau observer [8], has been constructed for a state estimation of non-linear systems. This observer relates the linear part and non-linear part of a dynamic system. The gain of Thau observer can be obtained if the synchronization error converges to a bounded zone [9].

Before to start the Thau observer design, is necessary to make a change of variable in system (2) in order to shift the equilibrium point to the origin of state-space and satisfy $g(0) = 0$. With $\bar{x}_1 = x_1 - x_1^*$ the system (2) can be rewritten as:

$$\dot{x} = Ax + g(x) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -c & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ b\sin(x_2 + 1) \end{bmatrix}; \tag{5}$$

with $x = [\bar{x}_1, x_2, x_3]$; $b = 4$ and $c = 2.2$.

Computing the Jacobi matrix of $g(x)$ we can obtain the Lipschitz constant ℓ, as result, we get:

$$\ell \leq ||g'(x)|| \leq b = 4. \tag{6}$$

We chose the output vector $y = [x_1, x_2]^T$. In this case the matrix $A_0 = A - KC$ of Thau observer is given by:

$$A_0 = \begin{bmatrix} -k_1 & 1-k_4 & 0 \\ -k_2 & -k_5 & 1 \\ -1-k_3 & -c-k_6 & -1 \end{bmatrix}. \tag{7}$$

We assign parameters of the feedback matrix as $k_3 = -1$; $k_4 = k_2 + 1$; $k_6 = -(c+1)$. The Thau observer will be asymptotically stable if A_0 is symmetric and satisfies [10]

$$\frac{||A_0||^{n-1}}{|det(A_0)|} \leq \frac{2}{\ell}. \tag{8}$$

Choosing $k_1 = 15$; $k_2 = 0$; $k_5 = 10$; and using (6) the inequality (8) can be satisfied. Hence the Thau observer is constructed:

$$\begin{aligned} \dot{\hat{\bar{x}}}_1 &= \hat{x}_2 + k_1(\bar{x}_1 - \hat{\bar{x}}_1) + k_4(x_2 - \hat{x}_2); \\ \dot{\hat{x}}_2 &= \hat{x}_3 + k_2(\bar{x}_1 - \hat{\bar{x}}_1) + k_5(x_2 - \hat{x}_2); \\ \dot{\hat{x}}_3 &= \pm b \sin(\pm \hat{x}_2 + 1) - \bar{x}_1 - c\hat{x}_2 - \hat{x}_3 \\ &\quad + k_3(\bar{x}_1 - \hat{\bar{x}}_1) + k_6(x_2 - \hat{x}_2). \end{aligned} \tag{9}$$

Numerical simulations have realized in order to show the synchronization of Thau observer and GS11 system, see Fig. 3 and 4.
The initial conditions of (5) is $[x_1(0), x_2(0), x_3(0)] = [-3, 0, 0]$ and the initial condition of Thau observer is $[\hat{x}_1(0), \hat{x}_2(0), \hat{x}_3(0)] = [0, 3, 3]$.

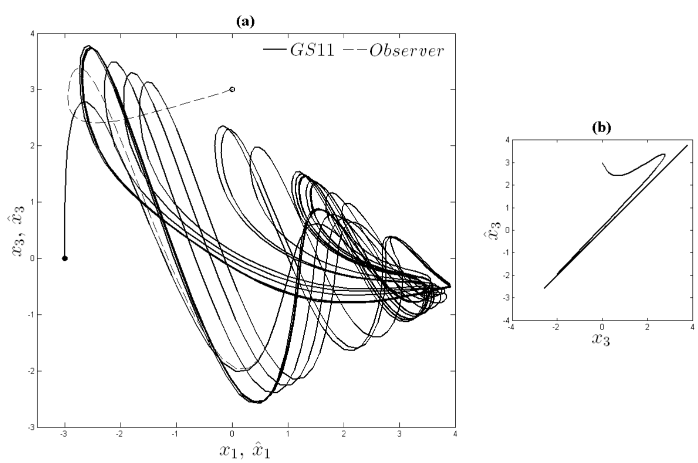

Fig. 3. Synchronization of GS systems:(a) Comparison of dynamics of GS system and Thau observer, (b) Synchronization between real states and estimated states.

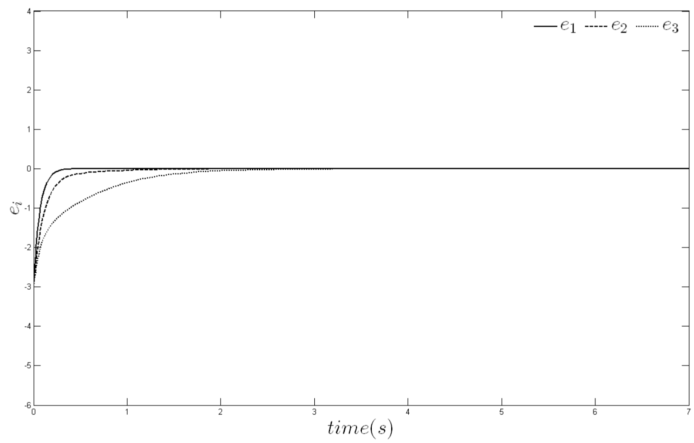

Fig. 4. Error dynamics of master-slave synchronization of Thau observer and GS11 system.

We can see in Fig. 3(a) that dynamic of (9) converge to chaotic attractor of (5) and in Fig. 1(b) that $\hat{x}_3$ is synchronized with x_3. Fig. 4. The error dynamics of master-slave synchronization converge to zero in approximately 4 seconds. This observer ensures asymptotic stability, but if we desire to speed up the convergence time is necessary to use some auxiliary technique.

4 Sliding-mode observer design for GS11 Sprott system

Sliding-mode observer is useful for many reasons: we have reduced observation error dynamics, for a finite-time convergence for all the observables states, to design under some conditions an observer for non-smooth system and robustness under parameter variations [21].

We propose the following observer for GS11 system:

$$\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + L_1 sign(y - \hat{x}_1);\\
\dot{\hat{x}}_2 &= \hat{x}_3 + L_2 sign(y - \hat{x}_1);\\
\dot{\hat{x}}_3 &= \pm b\sin(\pm\hat{x}_2 + 1) - \hat{x}_1 - c\hat{x}_2 - \hat{x}_3 - L_3 sign(y - \hat{x}_1);\\
y &= x_1;
\end{aligned} \tag{10}$$

with $L_1; L_2; L_3 > 0$.

The observer error is defined as $e = x - \hat{x}$. Dynamics of observer error is described by the following equation:

$$\begin{aligned}
\dot{e}_1 &= e_2 - L_1 sign(e_1);\\
\dot{e}_2 &= e_3 - L_2 sign(e_1);\\
\dot{e}_3 &= b\sin\theta \mp b\sin\hat{\theta} - e_1 - ce_2 - e_3 - L_3 sign(e_1);
\end{aligned} \tag{11}$$

where $\theta = x_2 + 1$; $\hat{\theta} = \hat{x}_2 + 1$.

In order to verify that trajectories of estimated states converge to real states, the following candidate Lyapunov function is proposed:

$$V(e) = \frac{1}{2}e^T Ae + [aL_1 + (ac+1)L_2 + L_3]\,|e_1| > 0 \tag{12}$$

where:

$$A = \begin{bmatrix} 1 & a & 0 \\ a & ac+1 & 1 \\ 0 & 1 & a \end{bmatrix}; \quad 1 < a < c; \quad e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}. \tag{13}$$

The time derivative along the trajectory of the system is:

$$\begin{aligned} \dot{V}(e) = &-(L_1 + aL_2)|e_1| - (c-a)e_2^2 - (a-1)e_3^2 - \beta L_1 \\ &+ b(e_2 + ae_3)(\sin\theta - \sin\hat{\theta}) - (L_2 + aL_3)sign(e_1)e_3; \end{aligned} \tag{14}$$

with $\beta = aL_1 + (ac+1)L_2 + L_3$.

Since $sin(x)$ and $sign(x)$ are bounded functions, then Eq. (27) can be expressed as:

$$\begin{aligned} \dot{V}(e) \leq &-(L_1 + aL_2)|e_1| - (c-a)e_2^2 - (a-1)e_3^2 - \beta L_1 \\ &+ 2b|e_2| + (2ab + L_2 + aL_3)|e_3| \end{aligned} \tag{15}$$

Using the norm properties $-x^T Qx < -\lambda_{\min}\{Q\}||x||_2^2$ and $||x||_1 \leq \sqrt{n}||x||_2$, $\dot{V}(e)$ can be written:

$$\begin{aligned} \dot{V}(e) \leq &-(L_1 + aL_2)|e_1| - \min\{(c-a), (a-1)\}||e_x||_2^2 \\ &+ \sqrt{2}\max\{2b, 2ab + L_2 + aL_3\}||e_x||_2 - \beta L_1 \end{aligned} \tag{16}$$

where $e_x = [|e_2|, |e_3|]$. If its possible to satisfy the following condition

$$\beta L_1 > \sqrt{2}\max\{2b, 2ab + L_2 + aL_3\}||e_x||_2; \tag{17}$$

we can guarantee local asymptotic stability. Finally, by applying Theorem 38 from [21] can be concluded that error dynamics converges to zero in finite-time.

We have realized numerical simulations in order to show the convergence of proposed sliding-mode to GS11 system as shwon Figs. 5 and 6. The initial condition of GS11 system is $[x_1(0), x_2(0), x_3(0)] = [0, 0, 0]$, the initial condition of proposed sliding-mode observer is $\hat{x}_1(0) = 3$ ($i = 1, 2, 3$) and $L_1 = 4$, $L_2 = 10$, $L_3 = 10$.

Fig. 5(a) shows that trajectories of sliding-mode observer converge to chaotic attractor of GS11 system and Fig. 5(b-c) show the synchronization between real states and estimated states. We can see Fig. 6 shows that error dynamics of master-slave synchronization converge to zero in approximately 8 seconds. This observer may only guarantee convergence in infinite time.

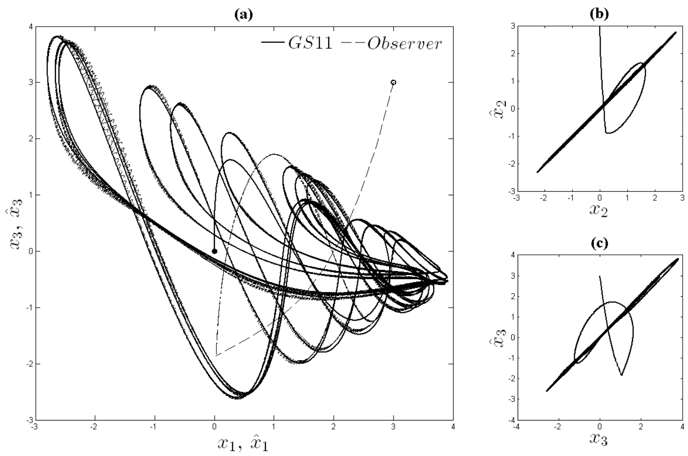

Fig. 5. Synchronization of GS systems: (a) Comparison of dynamics of GS system and sliding-mode observer, (b-c) Synchronization between real states and estimated states.

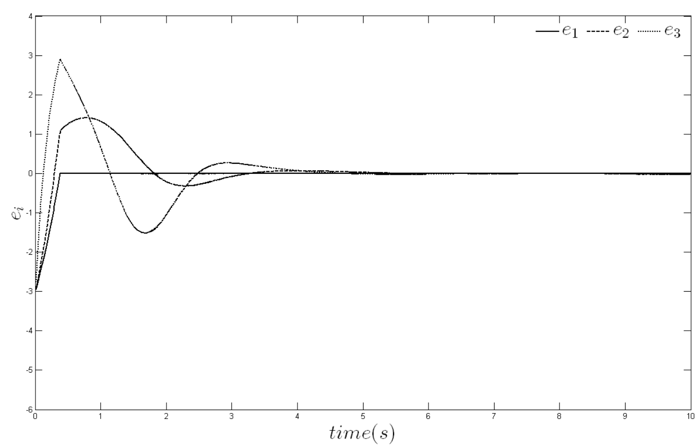

Fig. 6. Error dynamics of master-slave synchronization of Sliding-mode observer and GS11 system.

5 Robustness testing

Robustness test have realized in order to know the behaviour of proposed observer under variations of parameter and disturbances of the coupling signal. We made variations of parameter c of non-linear function of each observer and added a uniformly distributed noise (bound for both sides) in the coupling signal. The numerical simulations were carried out from the same initial conditions used to show the synchronization of observers and GS11 system in the previous sections.

5.1 Variations of parameter

We made variations of parameter c of non-linear function of each observer. The parameter variation performed was $c+\varepsilon$. The Table 2 shows the synchronization error rate of each observer, considering $\varepsilon = 0.1$ and $\varepsilon = 0.5$.

Table 2. Test of variation of parameter c.

	$\varepsilon = 0.1$	$\varepsilon = 0.5$
Linear injection	5.86%	13.11%
Thau observer	13.48%	29.77%
Slindig-mode observer	12.96%	27.97%

In this test we can not make variations of parameter b of non-linear term $f(x_2)$, this introduce a scaling factor of amplitude that implies a modification of the structure of the master-slave scheme.

We provide a discussion of results in the conclusions of this paper.

5.2 Disturbances of the coupling signal

The second test was to introduce a uniformly distributed noise δ (bound for both sides) in the coupling signal, in order to simulate disturbances in the communication channel. The Table 3 shows the synchronization error rate of each observer, considering $|\delta| \leq 0.5$ and $|\delta| \leq 1$.

Table 3. Perturbation test.

	$\|\delta\| \leq \|0.5\|$	$\|\delta\| \leq \|1\|$
Linear injection	27.89%	54.81%
Thau observer	6.12%	8.03%
Slindig-mode observer	5.39%	9.01%

A discussion about the results of this test are provided in the conclusion of this paper.

6 Conclusion

In this paper we have presented a three non-linear observers design in order to solve the chaos synchronization problem of some chaotic systems. Also robustness test have realized in order to know the behaviour under variation of parameters and disturbances in the coupling signal.

We designed a non-linear observer with linear output injection that guarantees asymptotic stability for all GS systems (GS1-GS12). This observer measures only one state variable. If its chosen an appropriate gain matrix we can modify transient response characteristics of the error convergence, e.g., damping, peak time, settling time. Also, we can use different non-linear terms listed in Table 1, this does not affect the observer design. In addition, a Thau observer is designed for GS11 system. This observer measures two state variables. To construct this observer is required to choose an appropriate feedback matrix K such that matrix A_0 is a stable and symmetric matrix and know the Lipschitz constant. Further, we designed a sliding-mode observer for GS11 system. This observer measures only one state variable. We can guarantee local asymptotic stability if a condition is satisfied and conclude that finite-time convergence of error dynamics by applying a theorem.

From the observer designs and robustness testing we notice that each proposed observer has benefits if they are applied in a scheme of secure communication system. The non-linear observer with output linear injection is a good alternative to implement in a scheme of secure communications systems with low noise levels. Presents robustness to variations of parameter. Also, we can modify the transient response characteristics of error convergence, according to requirements of communication channel and electronic circuit specifications of the system implementation. The Thau observer and sliding-mode observer are a good alternative to schemes of secure communications systems with disturbances in communication channel. Both observer have strong robustness to disturbances in communication channel. But, the measurement of two state variables of Thau observer may be a disadvantage. And the implementation of sliding-mode observer presents chattering phenomenon, then is necessary to find a method to reduce or eliminate it.

We consider as future work to implement the non-linear observer with linear output injection in some chaotic masking application with low noise levels, considering all cases of non-linear functions. Also, to implement the sliding-mode observer in a scheme of secure communication system with disturbances in communication channel.

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